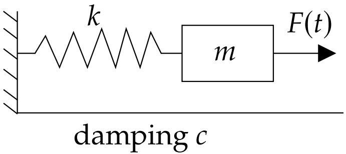
17. Mechanical vibrations, part 1: free undamped motion (Notes on Diffy Qs, 2.4)

Jiří Lebl

Oklahoma State University

The textbook: `https://www.jirka.org/diffyqs/`

Mass on a spring:



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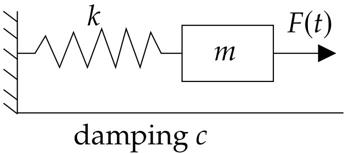

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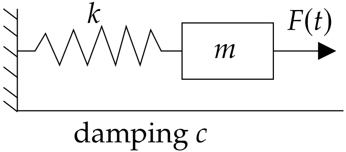

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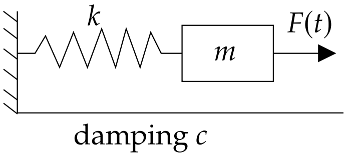

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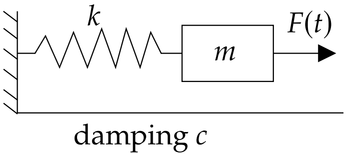

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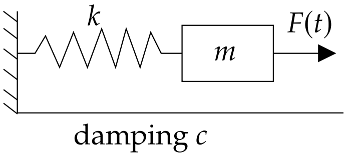

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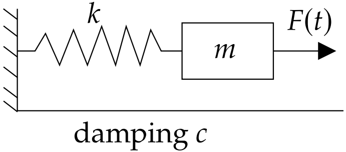

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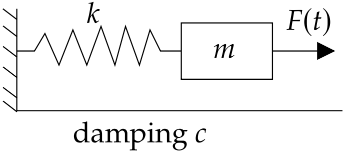

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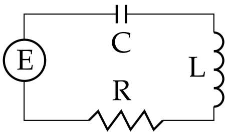
RLC circuit:



Resistor: R ohms.
Inductor: L henries.
Capacitor: C farads.
Electrical source: $E(t)$ volts.
$Q(t) =$ charge on the capacitor in coulombs.
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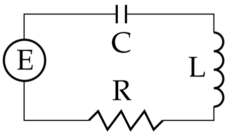

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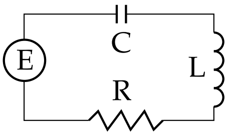

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Differentiate to get

$$LI''(t) + RI'(t) + \frac{1}{C}I(t) = E'(t).$$

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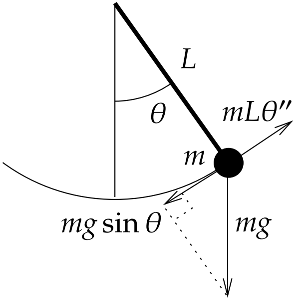

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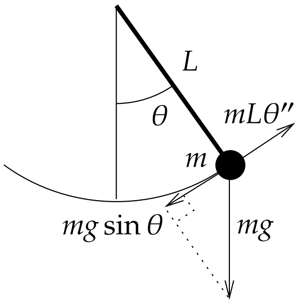

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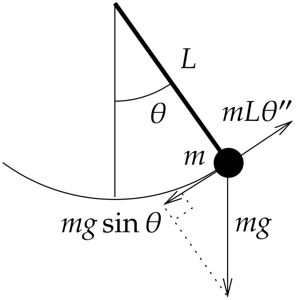

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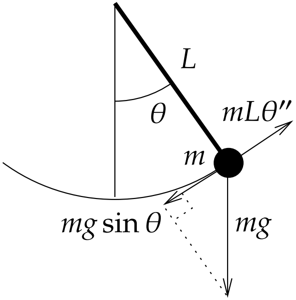

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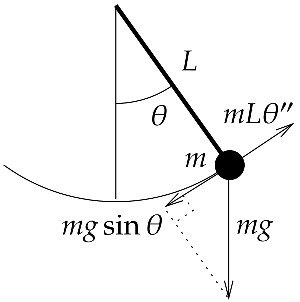

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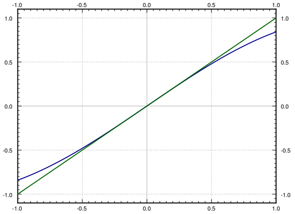

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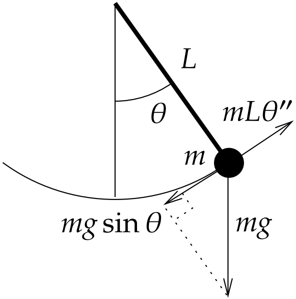

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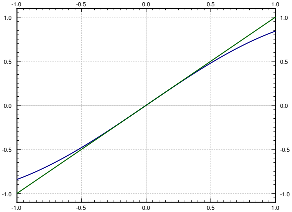

So as long as θ is small we have roughly:

$$\theta'' + \frac{g}{L}\theta = 0.$$

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Note: *period* is the reciprocal of *frequency* (time of one cycle): $\dfrac{2\pi}{\omega_0}$.

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The amplitude is $C = \sqrt{A^2 + B^2} = \sqrt{0.25 + 0.25} = \sqrt{0.5} \approx 0.707$ meters.

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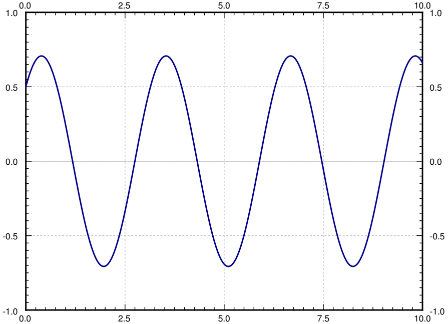
What is the frequency of the oscillation?
What is the amplitude?



The problem is
$2x'' + 8x = 0, \quad x(0) = 0.5, \quad x'(0) = 1.$

Angular frequency $\omega_0 = \sqrt{k/m} = \sqrt{4} = 2.$

Frequency in Hertz: $2/2\pi = 1/\pi \approx 0.318.$

The general solution is: $x(t) = A\cos(2t) + B\sin(2t).$
$x(0) = 0.5 \quad \Rightarrow \quad A = 0.5.$
$x'(t) = -2(0.5)\sin(2t) + 2B\cos(2t). \quad x'(0) = 1 \quad \Rightarrow \quad B = 0.5.$

The amplitude is $C = \sqrt{A^2 + B^2} = \sqrt{0.25 + 0.25} = \sqrt{0.5} \approx 0.707$ meters.

Solution is: $\quad x(t) = 0.5\cos(2t) + 0.5\sin(2t)$

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where $x(0) = A$ and $x'(0) = \omega_0 B$.

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Calculators and computer software often have a function like `atan2(A,B)` that computes γ.