4. Integrals as solutions (Notes on Diffy Qs, 1.1)

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The textbook: `https://www.jirka.org/diffyqs/`

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So solve for y!

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If we let C be arbitrary, then $y = Ce^{kx}$ includes all the possibilities, including $y = 0$.

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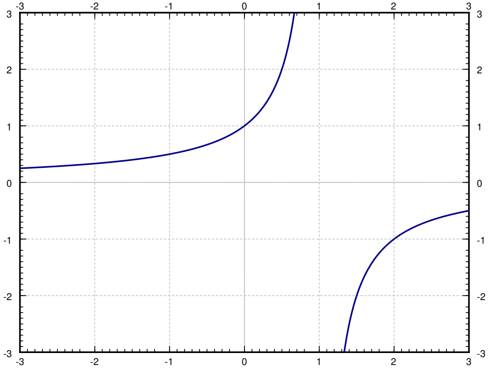

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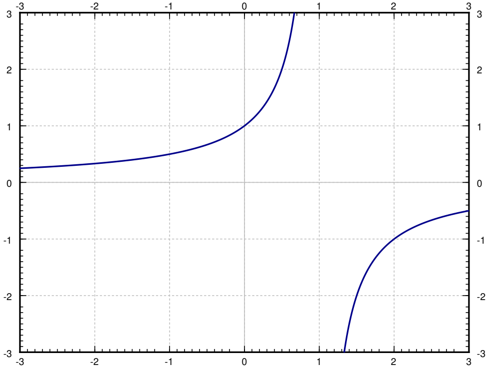

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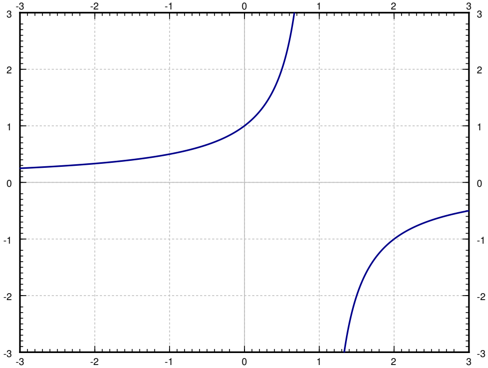

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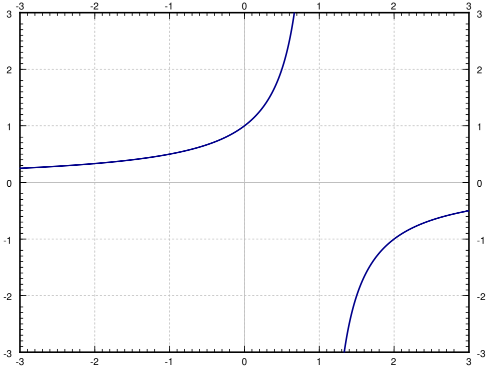

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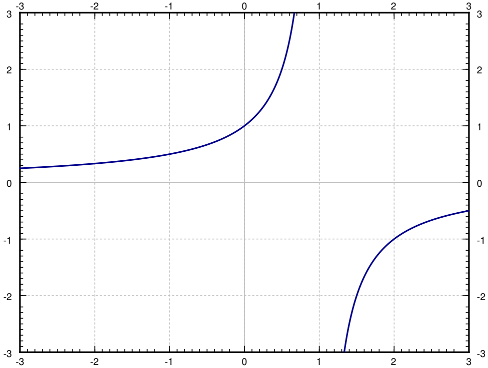

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Solution: $\quad x(t) = 2e^{t/2} - 2$.

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Example: A car drives at a speed of $e^{t/2}$ meters per second, where t is time in seconds. How far did the car get in 2 seconds (starting at $t = 0$)? How far in 10 seconds?

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Exercise

Solve for v, and then solve for x. Find $x(10)$ to answer the question.