BA: 0.3 part 1

Jiří Lebl

Departemento pri Matematiko de Oklahoma Ŝtata Universitato

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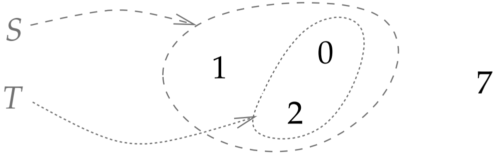

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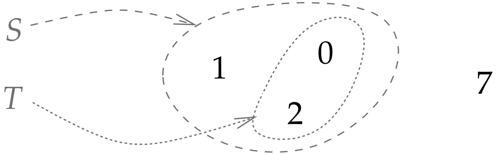

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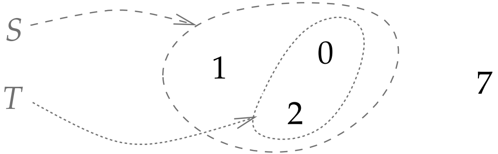

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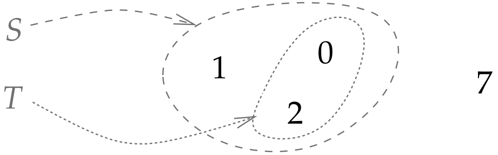

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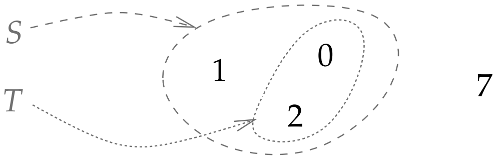

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Note: $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$.

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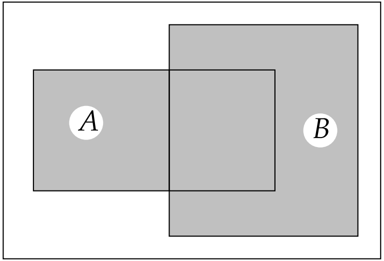
(v) A and B are *disjoint* if $A \cap B = \emptyset$.



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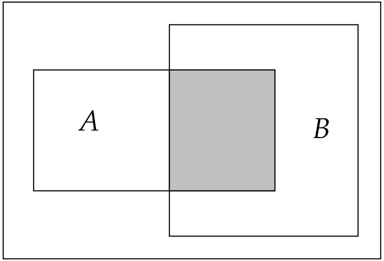

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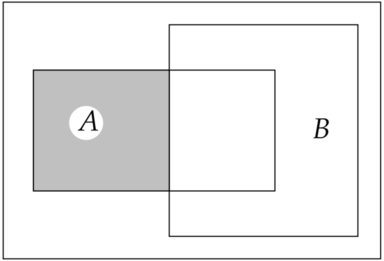

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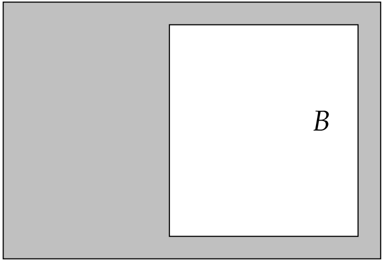

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Theorem (DeMorgan)

Let A, B, C be sets. Then $\quad (B \cup C)^c = B^c \cap C^c, \quad (B \cap C)^c = B^c \cup C^c,$

or, $\quad A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C), \quad A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C).$

Proof: The second statement proves the first if A is the "universe."

Let us prove $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ (remember the definition of =).

First, suppose $x \in A \setminus (B \cup C)$.
Then x is in A, but not in B nor C.
Hence x is in A and not in B, that is, $x \in A \setminus B$.
Similarly $x \in A \setminus C$.
Thus $x \in (A \setminus B) \cap (A \setminus C)$.

On the other hand, suppose $x \in (A \setminus B) \cap (A \setminus C)$.
In particular, $x \in (A \setminus B)$, so $x \in A$ and $x \notin B$.
As also $x \in (A \setminus C)$, then $x \notin C$.
Hence $x \in A \setminus (B \cup C)$.

The proof of the other equality is left as an exercise. $\square$

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It is not hard to see that the order of the unions can be swapped.
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The index set could be another set than natural numbers in which case we use the following notation:

$$\bigcup_{\lambda \in I} A_\lambda \coloneqq \{x : x \in A_\lambda \text{ for some } \lambda \in I\}, \qquad \bigcap_{\lambda \in I} A_\lambda \coloneqq \{x : x \in A_\lambda \text{ for all } \lambda \in I\}.$$