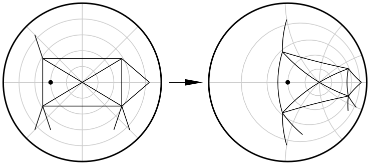
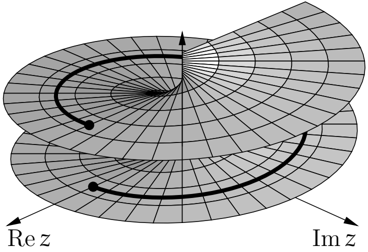
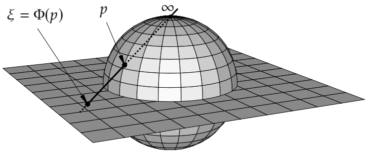
Cultivating Complex Analysis: Introduction

Jiří Lebl

Departemento pri Matematiko de Oklahoma Ŝtata Universitato







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My goal is to record a lecture every week or two for the forseeable future.

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No courses that are usually considered graduate courses are required. Explicitly, Lebesgue integral and measure theory is not needed.

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- Montel and Riemann (chapter 6)
  - Arzelà–Ascoli, Montel, Riemann mapping theorem.

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- Analytic continuation (chapter 10)
  - Schwarz reflection principle, analytic continuation along paths, Monodromy theorem.

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Complex differentiable versus (real) differentiable

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$(z, \bar{z})$ versus (x, y)