Cultivating Complex Analysis: Meromorphic functions (5.2.4)

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To emphasize we will often say "Let $f\colon U \to \mathbb{C}_\infty$ be meromorphic."

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Exercise: Show that an injective holomorphic $f \colon \mathbb{C}_\infty \to \mathbb{C}_\infty$ is an LFT.