

17. Mechanical vibrations, part 1: free undamped motion (Notes on Diffy Qs, 2.4)

Jiří Lebl

Oklahoma State University

The textbook: <https://www.jirka.org/diffyqs/>

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t = time in seconds.

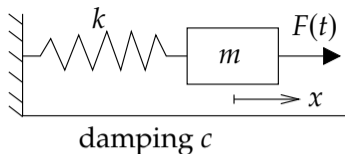
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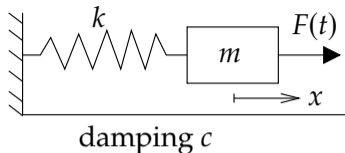
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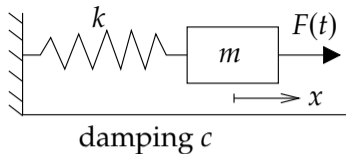
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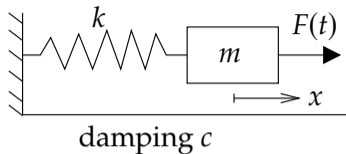
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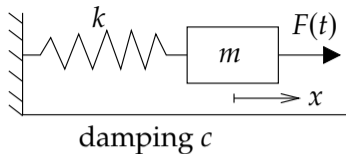
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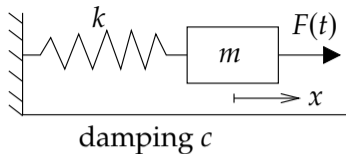
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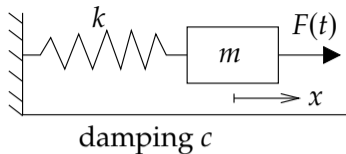
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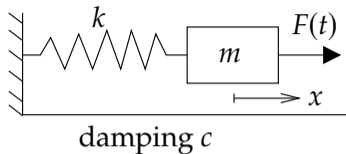
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damped if $c > 0$, and *undamped* if $c = 0$.

RLC circuit:

Resistor: R ohms.

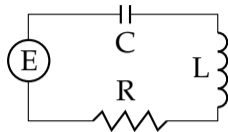
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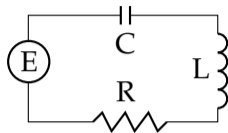
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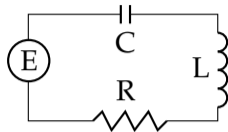
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Differentiate to get

$$LI''(t) + RI'(t) + \frac{1}{C}I(t) = E'(t).$$



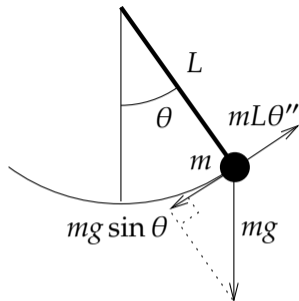
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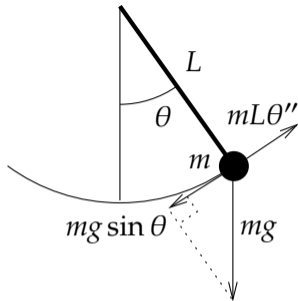
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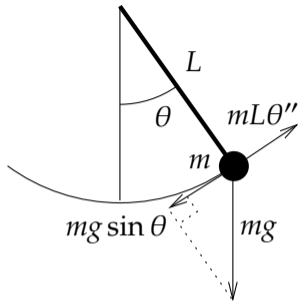
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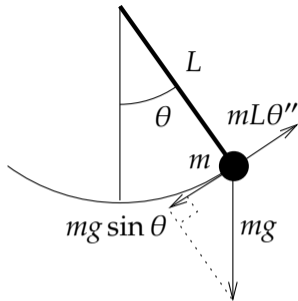
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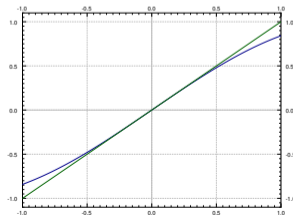
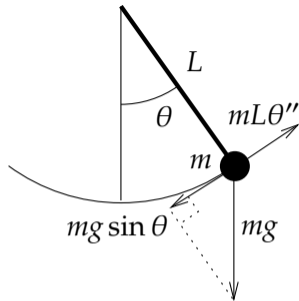
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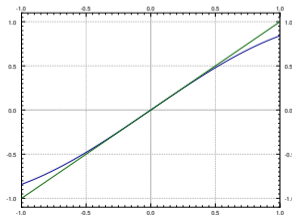
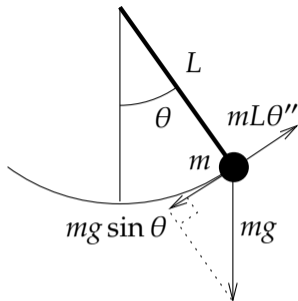
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So as long as θ is small we have roughly:

$$\theta'' + \frac{g}{L} \theta = 0.$$



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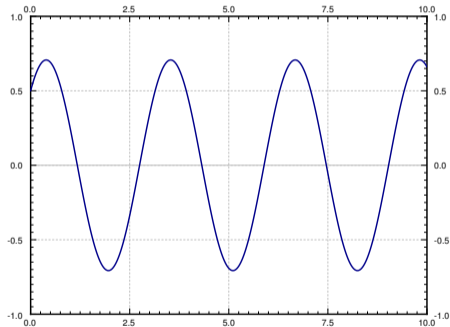
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In the example $\tan \gamma = 1$. $\arctan 1 = \pi/4$.

For free undamped motion, solution is

$$x(t) = A \cos(\omega_0 t) + B \sin(\omega_0 t),$$

where $x(0) = A$ and $x'(0) = \omega_0 B$.

Amplitude $C = \sqrt{A^2 + B^2}$.

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Calculators and computer software often have a function like $\text{atan2}(A, B)$ that computes γ .