

## 4. Integrals as solutions (Notes on Diffy Qs, 1.1)

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The textbook: <https://www.jirka.org/diffyqs/>

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So solve for  $y$ !

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If we let  $C$  be arbitrary, then  $y = Ce^{kx}$  includes all the possibilities, including  $y = 0$ .

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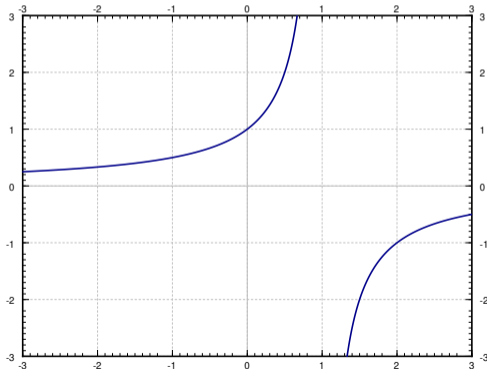
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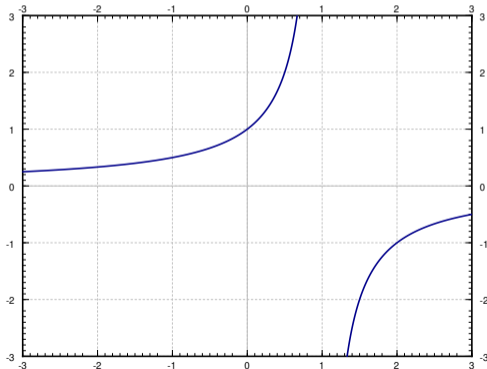
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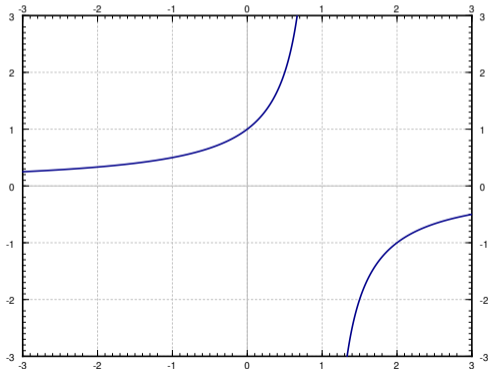
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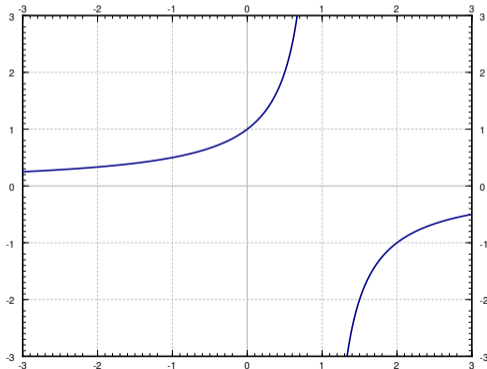
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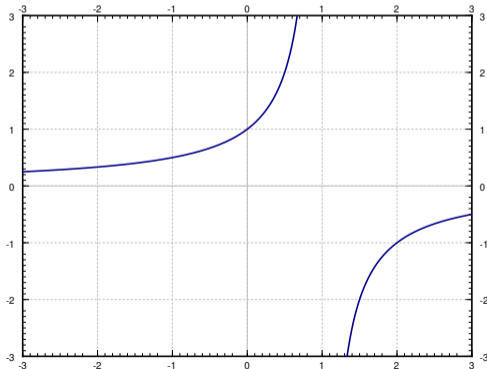
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### Exercise

*Solve for  $v$ , and then solve for  $x$ . Find  $x(10)$  to answer the question.*